Statistics of delay times in mesoscopic systems as a manifestation of eigenfunction fluctuations

Alexander Ossipov[1, *] and Yan V. Fyodorov[2, †]

[1] *The Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, 34014 Trieste, Italy*
[2] *Department of Mathematical Sciences, Brunel University, Uxbridge UB83PH, United Kingdom*
(Dated: February 2, 2008)

We reveal a general explicit relation between the statistics of delay times in one-channel reflection from a mesoscopic sample of any spatial dimension and the statistics of the eigenfunction intensities in its closed counterpart. This opens a possibility to use experimentally measurable delay times as a sensitive probe of eigenfunction fluctuations. For the particular case of quasi-one dimensional geometry of the sample we use an alternative technique to derive the probability density of partial delay times for any number of open channels.

I. INTRODUCTION

The standard way of describing scattering in quantum systems employs a $M \times M$ unitary matrix known as S-matrix, with M standing for the number of scattering channels open at a given energy. Then the Wigner delay time[1] defined as the energy derivative of the total scattering phaseshift: $\tau_w = -i\hbar \frac{\partial}{\partial E} \ln \mathrm{Det} S$ is one of the most important and frequently used characteristics of quantum scattering, see e.g.[2,3]. It can be interpreted as a time delay in propagation of the peak of the wave packet due to scattering in comparison with a free wavepacket propagation. More detailed characterization of scattering calls for introducing the Wigner-Smith time delay matrix[4] $Q = i\hbar \frac{\partial S^{\dagger}}{\partial E} S$, whose eigenvalues are frequently called *proper* delay times. Alternatively, denoting M unimodular eigenvalues of the $S-$matrix as $\{\exp(i\theta_1), \ldots, \exp(i\theta_M)\}$, one can consider *partial* delay times defined[5] as $\tau_a = \partial \theta_a / \partial E$, $a = 1, \ldots, M$. Beyond one-channel case proper and partial delay times differ, although the sum of partial/proper delay times over all M scattering channels are always equal and yield the Wigner delay time.

Statistics of delay times of all sorts were studied intensively in the framework of quantum chaotic scattering. Earlier works on the subject used various approximation schemes, frequently not very well controlled[6]. It is however well known that quantum statistical properties of classically chaotic systems are to large extent universal and independent of their microscopic nature[7,8] which allows one to use the Random Matrix Theory (RMT) for description of their properties[9]. When applied to the scattering problems and in particular to the study of delay times statistics the RMT yielded many exact analytical results in recent years, see[5,10,11,12,13,14,15,16,17,18] and references therein. Those were successfully verified in numerical simulations of several quantum chaotic systems of quite a diverse nature, see e.g.[19]. Some properties of delay times were also studied by semiclassical methods[20].

In the general framework of mesoscopic systems the RMT regime corresponds to the so called zero-dimensional limit. In this limit the closed counterpart of a scattering system is characterized by the only non-trivial energy scale which is simply the mean level spacing Δ, and RMT predictions for statistics of eigenfunctions and energy levels were successfully tested experimentally in microwave and acoustic experiments[21].

Beyond the zero-dimensional limit the particular nature of the sample geometry and type of disorder causing chaotic behaviour starts to play an important role. In particular, in systems with random short-range impurities another energy scale known as the Thouless energy E_c plays a prominent role, the ratio $g = 2\pi E_c/\Delta$ being known as the dimensionless conductance of the sample. This parameter controls the system properties, with the universal RMT regime being recovered in the limit $g \to \infty$. The statistical properties of the energy spectra and eigenfunctions of closed mesoscopic systems as a function of dimensional conductance were under intensive investigation for more than a decade, both analytically[22,23,24,25,26], numerically[27,28], and some predicted features are also seen in microwave experiments[29]. The research resulted in a detailed picture emerging for various regimes, including the most difficult case of the Anderson localization transition, see the review[30] for a general picture. At the same time systematic statistical analysis of delay times beyond the universal zero-dimensional limit is still lacking, apart from a few useful insights in[31,32,33,34,35,36].

The goal of the present paper is to initiate systematic investigation of delay times beyond the zero-dimensional approximation in metallic, critical and localized regimes. To this end, in the first part of the paper we reveal a very general relation between the delay time distribution and the distribution of eigenfunction intensities for a single channel scattering. On one hand, this relation allows us to use the existent knowledge on eigenfunction statistics to provide explicit expressions for delay times distributions in various regimes of interest. On the other hand, since phase shifts

and delay times are experimentally measurable quantity, especially in the one-channel reflection experiment[37,38,39,40], the relation opens a new possibility for experimental study of eigenfunctions.

In the second part of the paper we consider a model of a quasi-one-dimensional disordered wire represented by Banded Random Matrix (BRM) ensemble. It is well-known that in the limit of the thick wire this model is exactly solvable[44] by the transfer matrix approach[45]. The same technique when combined with methods introduced in[5] for the scattering problem yields the distribution of partial delay times for any finite number of open channels. In particular, for the single open channel it reproduces the result obtained from the general relation developed in the first section.

II.   DELAY TIME DISTRIBUTION FOR ONE-CHANNEL SCATTERING: RELATION TO EIGENFUNCTION STATISTICS.

Consider a single-channel antenna/lead perfectly coupled to an absorptive disordered system, characterized by the (energy, absorption and position dependent) reflection coefficient $R = R(\eta, \mathbf{r}) \equiv |S(E, \eta)|^2 \leq 1$, where $\mathbf{r}$ stands for the coordinate of antenna port, and $\eta \geq 0$ for the absorption parameter. Here and henceforth the absorption is considered as uniform, position-independent process and therefore can be accounted for as an imaginary shift of the energy of incoming waves. For the definition of "perfect coupling" see the equation Eq. (57) below and the discussion preceding it.

The Wigner delay time can be conveniently written as $\tau_w = \lim_{\eta \to 0} \frac{1-R}{2\eta}$, see e.g.[40]. Introduce variable $x \geq 1$ via $R = \frac{x-1}{x+1}$, and let $\mathcal{P}_0^{(\eta)}(x)$ stands for the distribution function of x at given absorption.

In the limit of zero absorption $\eta \to 0$ the reflection coefficient $R \to 1$, hence $x \to \infty$. In fact, correct limit is such that $x = \frac{z}{\eta}$, $z < \infty$. The variable z is nothing else but the inverse delay time:

$$\frac{1}{\tau_w} = \lim_{\eta \to 0} \frac{2\eta}{1-R} = \lim_{\eta \to 0} \eta(x+1) = z \tag{1}$$

The distribution function function $\mathcal{P}_0^{(\eta)}(x)$ behaves in the limit of vanishing absorption as $\tilde{\mathcal{P}}_0(z/\eta) \to \eta \tilde{\mathcal{P}}_0(z)$, where $\tilde{\mathcal{P}}_0(z)$ is a well-defined distribution function of variable z (i.e. of the inverse delay time).

From the other hand, let $G(E + i\eta; \mathbf{r}; \mathbf{r}')$ be the Green's function of a closed system with broadening η, so that the local density of states (LDoS) at any point $\mathbf{r}$ in the sample is given by

$$\tilde{v}_\eta(\mathbf{r}) = -\frac{1}{\pi} \text{Im} \, G(E + i\eta; \mathbf{r}; \mathbf{r}) = \frac{1}{\pi} \sum_{n=1}^{N} |\psi_n(\mathbf{r})|^2 \frac{\eta}{(E - E_n)^2 + \eta^2} \tag{2}$$

Here $\psi_n(\mathbf{r})$ stands for the local amplitude of the eigenfunction of the wave/Schrödinger operator describing a classical wave/quantum particle in the disordered sample and corresponding to the eigenfrequency/energy level E_n.

Using this relation it is easy to satisfy oneself that for any integer $k = 1, 2, 3, \ldots$ holds

$$\lim_{\eta \to 0} [\eta^{k-1} \pi^k \tilde{v}_\eta^k(\mathbf{r})] = \frac{\sqrt{\pi} \Gamma(k - \frac{1}{2})}{\Gamma(k)} \sum_{n=1}^{N} |\psi_n(\mathbf{r})|^{2k} \delta(E - E_n) \tag{3}$$

where $\Gamma(z)$ stands for the Euler gamma-function.

Indeed, in the limit $\eta \to 0$ the nonvanishing contribution to the above expression comes only from the most singular ("diagonal") term in the product of sums over eigenvalues:

$$\eta^{k-1} \sum_{n=1}^{N} |\psi_n(\mathbf{r})|^{2k} \frac{\eta^k}{[(E - E_n)^2 + \eta^2]^k}$$

Evaluating the standard integrals, we observe that:

$$\int_{-\infty}^{\infty} dx \frac{\eta^{2k-1}}{[x^2 + \eta^2]^k} = \int_{-\infty}^{\infty} du \frac{1}{[u^2 + 1]^k} = \frac{\sqrt{\pi} \Gamma(k - \frac{1}{2})}{\Gamma(k)}$$

which yields the above result Eq.(3).

The right hand side of Eq.(3) is proportional to the moments of the local eigenfunction intensity $y = V|\psi_n(\mathbf{r})|^2$, with V standing for the volume of our sample, averaged in a small energy window around point E of the spectrum. Denoting additional disorder averaging by brackets and introducing the distribution function $\mathcal{P}_v(v)$ of the random

variable $v_\eta(\mathbf{r}) = \tilde{v}_\eta(\mathbf{r}) V \Delta$, with Δ being the mean level spacing of the sample, we therefore can rewrite our relation as:

$$\langle y^k \rangle = \frac{\sqrt{\pi} \Gamma(k)}{\Gamma(k - \frac{1}{2})} \lim_{\eta \to 0} \left(\frac{\eta \pi}{\Delta} \right)^{k-1} \int_0^\infty dv \, v^k \, \mathcal{P}_v(v) \tag{4}$$

At the next step we use a recently discovered relation between the probability distribution $\mathcal{P}_v(v)$ of LDoS and the function $\mathcal{P}_0^{(\eta)}(x), \ x \geq 1$[46]:

$$\mathcal{P}_v(v) = \frac{\sqrt{2}}{\pi v^{3/2}} \int_0^\infty dq \, \mathcal{P}_0^{(\eta)} \left[q^2 + \frac{1}{2} \left(v + \frac{1}{v} \right) \right] , \tag{5}$$

valid for those disordered and chaotic systems which allow an effective description in terms of the nonlinear $\sigma-$ model. The existence of this relation is based on the two following observations. First, the scattering matrix for a one-channel scattering can be expressed in terms of the diagonal matrix element of the resolvent of the corresponding closed system (see Eqs.(23,30) in the next section). In this way one can relate the probability distribution of LDoS and statistical properties of the scattering matrix. The second observation is that under conditions of perfect coupling the phase of the S-matrix is statistical independent of the S-matrix modulus $R = \frac{x-1}{x+1}$ and is distributed uniformly over the unit circle. Consequently, all non-trivial information on the scattering matrix is contained in the distribution function of its modulus R, or equivalently in the distribution function of x. This fact explains the appearance of the function $\mathcal{P}_0^{(\eta)}(x)$ on the right-hand side of Eq.(5) , see[46] for the detailed derivation and discussion. It is important also to remember that the above distribution $\mathcal{P}_v(v)$ is normalized in such a way that the first moment is equal to unity: $\langle v \rangle \equiv 1$.

Rescaling the variable $v = z/\eta$ in the integral (4) we see that it is $\lim_{\eta \to 0} \mathcal{P}_v(z/\eta)/\eta^2$ which we are interested in. The limit can be calculated from (5) by changing the variable in the integral to $q = t/\sqrt{\eta}$:

$$\lim_{\eta \to 0} \frac{1}{\eta^2} \mathcal{P}_v \left(\frac{z}{\eta} \right) = \frac{\sqrt{2}}{\pi z^{3/2}} \int_0^\infty dt \, \tilde{\mathcal{P}}_0 \left(t^2 + \frac{z}{2} \right) \tag{6}$$

In this way we bring Eq.(4) to the form

$$\langle y^k \rangle = \left(\frac{\pi}{\Delta} \right)^{k-1} \sqrt{\frac{2}{\pi}} \frac{\Gamma(k)}{\Gamma(k - \frac{1}{2})} \int_0^\infty dz \, z^{k-3/2} \int_0^\infty dt \, \tilde{\mathcal{P}}_0 \left(t^2 + \frac{z}{2} \right) \tag{7}$$

Introducing the new variable $s = t^2 + z/2$ we further notice that

$$\int_0^\infty dz \, z^{k-3/2} \int_0^\infty dt \, \tilde{\mathcal{P}}_0 \left(t^2 + \frac{z}{2} \right) = \int_0^\infty ds \, \tilde{\mathcal{P}}_0(s) \int_0^{2s} dz \, \frac{z^{k-3/2}}{2\sqrt{s - \frac{1}{2}z}} =$$

$$= \sqrt{\frac{\pi}{2}} \frac{\Gamma(k - \frac{1}{2})}{\Gamma(k)} \int_0^\infty ds \, \tilde{\mathcal{P}}_0(s) (2s)^{k-1} \tag{8}$$

Substituting now the last expression into (7) we arrive at the relation:

$$\int_0^\infty dy \, \mathcal{P}_y(y) \, y^k \equiv \langle y^k \rangle = \int_0^\infty ds \, \tilde{\mathcal{P}}_0(s) \left(\frac{2\pi s}{\Delta} \right)^{k-1} \tag{9}$$

valid for all integer $k \geq 1$. We therefore conclude that the distribution functions in the left and right hand sides of this expression are simply related to each other by $\mathcal{P}_y(\tilde{z}) = \tilde{\mathcal{P}}_0(\tilde{z}) \, \tilde{z}^{-1}$, where $\tilde{z} = 2\pi z/\Delta$. Remembering interpretation of z as the inverse delay time and introducing the distribution $\tilde{\mathcal{P}}_w(\tilde{\tau}_w)$ of scaled time delays $\tilde{\tau}_w = \tau \Delta/2\pi$ we finally arrive at the following simple but fundamental relation between moments of eigenfunction intensity y and the inverse moments of the time delay:

$$\langle \tilde{\tau}_w^{-k} \rangle = \langle y^{k+1} \rangle \tag{10}$$

resulting in the functional relation between the two distributions:

$$\tilde{\mathcal{P}}_w(\tilde{\tau}_w) = \frac{1}{\tilde{\tau}_w^3} \mathcal{P}_y \left(\frac{1}{\tilde{\tau}_w} \right) , \tag{11}$$

and constituting one of the central results of the present paper. Some remarks on the conditions of validity of Eqs.(10, 11) are given in the last section of the present paper.

Let us now demonstrate how the relation Eq.(11) works in various situations. Let us start with the case of a "zero-dimensional" system which can be described by the RMT. The distributions of eigenfunction intensities typical for the zero-dimensional case and various symmetry classes characterized by $\beta = 1, 2, 4$[9] are given by the so-called $\chi-$squared distribution:

$$\mathcal{P}_y(y) = C_\beta y^{\beta/2-1} e^{-\frac{\beta}{2}y}, \quad C_\beta = (\beta/2)^{\beta/2}/\Gamma(\beta/2) \tag{12}$$

with particular case $\beta = 1$ frequently being referred to as the Porter-Thomas distribution. The known expressions[11,12] for all three pure symmetry classes immediately follow as:

$$\mathcal{P}_w(\tilde{\tau}_w) = [(\beta/2)^{\beta/2}/\Gamma(\beta/2)]\tilde{\tau}_w^{-\beta/2-2} e^{-\beta/2\tilde{\tau}_w} \tag{13}$$

Moreover, the distribution of the one-channel delay times in the crossover regime between unitary ($\beta = 2$) and orthogonal ($\beta = 1$) symmetry classes was calculated in Ref.[13] :

$$\mathcal{P}_w(\tilde{\tau}_w) = \frac{1}{2\tilde{\tau}_w^3} \int_{-1}^1 d\lambda \int_1^\infty d\lambda_2\, \lambda_2^2 e^{-X^2(\lambda_2^2-1)} e^{-\lambda_2^2/\tilde{\tau}_w} I_0\left[\frac{\lambda_2\sqrt{\lambda_2^2-1}}{\tilde{\tau}_w}\right] \mathcal{T}_2(\lambda, \lambda_2), \tag{14}$$

$$\mathcal{T}_2(\lambda, \lambda_2) = 2X^2\left[(1-\lambda^2)e^{-\alpha} + \lambda_2^2(1-e^{-\alpha})\right] - (1 - e^{-\alpha}), \tag{15}$$

where $\alpha = X^2(1 - \lambda^2)$, $I_0(z)$ stands for the modified Bessel function, and X is a crossover driving parameter[41]. This result can be easily recovered from the distribution of the eigenfunction intensities in the crossover found in Ref.[47]:

$$\mathcal{P}_y(y) = 2\int_1^\infty dt\, \left\{\Phi_1(X) + \left[(tX)^2 - 1\right]\Phi_2(X)\right\}(tX)^2 I_0(yt\sqrt{t^2-1})e^{-yt^2 - X^2(t^2-1)}, \tag{16}$$

$$\Phi_1(X) = \frac{e^{-X^2}}{X}\int_0^X dx\, e^{x^2}, \qquad \Phi_2(X) = \frac{1 - \Phi_1(X)}{X^2}.$$

In the metallic regime beyond RMT the perturbative corrections to the body of the distribution of eigenfunction intensities were calculated using supersymmetric non-linear σ-model in Ref.[24]. Then relation (11) yields the following distributions of the Wigner delay times:

$$\mathcal{P}_w(\tilde{\tau}_w) = \frac{e^{-1/2\tilde{\tau}_w}}{\sqrt{2\pi}\tilde{\tau}_w^{5/2}}\left[1 + \frac{\kappa}{2}\left(\frac{3}{2} - \frac{3}{\tilde{\tau}_w} + \frac{1}{2\tilde{\tau}_w^2}\right) + \ldots\right] \quad \beta = 1,$$

$$\mathcal{P}_w(\tilde{\tau}_w) = \frac{e^{-1/\tilde{\tau}_w}}{\tilde{\tau}_w^3}\left[1 + \frac{\kappa}{2}\left(2 - \frac{4}{\tilde{\tau}_w} + \frac{1}{\tilde{\tau}_w^2}\right) + \ldots\right] \quad \beta = 2,$$

$$\mathcal{P}_w(\tilde{\tau}_w) = \frac{4e^{-2/\tilde{\tau}_w}}{\tilde{\tau}_w^4}\left[1 + \frac{\kappa}{2}\left(3 - \frac{6}{\tilde{\tau}_w} + \frac{2}{\tilde{\tau}_w^2}\right) + \ldots\right] \quad \beta = 4, \tag{17}$$

Here the parameter $\kappa = a/g$ is just the diagonal part of the so-called diffusion propagator $\Pi(\mathbf{r}, \mathbf{r})$[24,30] and is inversely proportional to the dimensionless conductance g. The exact value of the constant a depends essentially on the sample geometry and on the coordinates of the lead.

Eq.(17) holds for relatively large delay times $\tilde{\tau}_w \gtrsim \sqrt{\kappa}$, while in the opposite case the distribution is dominated by the existence of the anomalously localized states (see[30] for a review) and has the following behavior for dimensionality $d = 2, 3$[23]:

$$\mathcal{P}_w(\tilde{\tau}_w) \sim \exp\left(\frac{\beta}{2}\left\{-\frac{1}{\tilde{\tau}_w} + \kappa\frac{1}{\tilde{\tau}_w^2} + \ldots\right\}\right), \quad \kappa \lesssim \tilde{\tau}_w \lesssim \sqrt{\kappa}, \tag{18}$$

$$\mathcal{P}_w(\tilde{\tau}_w) \sim \exp(-C_d \ln^d(1/\tilde{\tau}_w)), \quad \tilde{\tau}_w \lesssim \kappa. \tag{19}$$

It deserves mentioning that the log-normal distribution found in Ref.[35] was observed for the opposite limit of *large* delay times and *many* open channels.

Another important consequence of (11) is that the eigenfunction multifractality typical for the vicinity of the Anderson localization transition[30] reflects itself in the distribution of Wigner delay times. Generally this means that the negative moments of the delay time scale anomalously with the system size L:

$$\left\langle\frac{1}{\tau_w^{q-1}}\right\rangle \sim L^{-D_q(q-1)} \tag{20}$$

where D_q is a fractal dimension of the eigenfunctions. The idea of the anomalous scaling (20) of the time delays was suggested by one of the authors in Ref.[36].

The fractal dimensions of the eigenfunctions is known analytically for a two-dimensional system in the metallic regime, where the deviation from the normal scaling is determined by the inverse conductance (the regime of weak multifractality)[30]:

$$\left\langle \frac{1}{\tau_w^{q-1}} \right\rangle \sim L^{-(2-\frac{q}{\beta\pi g})(q-1)}, \quad q \ll 2\beta\pi g. \tag{21}$$

another case when the anomalous dimensions are known analytically is the model of power-law random banded matrices, whose elements are independent random variables H_{ij} with the variance decreasing in a power-law fashion: $\left\langle (H_{ij})^2 \right\rangle = [1 + (|i-j|/b)^{2\alpha}]^{-1}$. For $\alpha = 1$ this model shows critical behavior and the fractal dimensions of the eigenfunctions can be calculated for $b \gg 1$[48]:

$$\left\langle \frac{1}{\tau_w^{q-1}} \right\rangle \sim L^{-(1-\frac{q}{2\beta\pi b})(q-1)}. \tag{22}$$

III. PARTIAL DELAY TIMES IN A QUASI-ONE-DIMENSIONAL GEOMETRY

In this section we consider a quasi-one-dimensional disordered sample of length L with a perfect lead attached to one of its edges, the other edge being impenetrable for the waves. The lead supports M scattering channels, thus the S-matrix is $M \times M$ unitary matrix and the partial delay times are defined in the standard way as $\tau_a = \partial\theta_a/\partial E$, $a = 1, \ldots, M$.

Following the method suggested for the zero-dimensional case in[5,11] we address the statistics of the partial delay times by considering the two-point correlation function of eigenvalue densities of the K-matrix. The latter matrix is Hermitian and is defined in terms of the scattering matrix via the relation

$$\hat{S} = \frac{\hat{I} - i\pi\hat{K}}{\hat{I} + i\pi\hat{K}}. \tag{23}$$

Denoting the real eigenvalues of the K-matrix by z_a, $a = 1, \ldots, M$ the partial delay times can be obviously written as $\tau_a(E) = \partial\theta_a(E)/\partial E = -\frac{2}{1+z_a^2(E)}\frac{\partial z_a(E)}{\partial E}$. Knowing the joint probability density of random variables z_a and $v_a = \partial z_a/\partial E$

$$\mathcal{P}_E(z,v) = \frac{1}{M}\left\langle \sum_{a=1}^{M} \delta(z-z_a)\,\delta\left(v - \frac{\partial z_a}{\partial E}\right)\right\rangle \tag{24}$$

one can recover the mean density of the partial delay times as

$$\mathcal{P}_\tau(\tau) = \frac{1}{M}\left\langle \sum_{a=1}^{M} \delta(\tau - \tau_a)\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dz\,dv\,\mathcal{P}_E(z,v)\,\delta\left(\tau + \frac{2v}{1+z^2}\right). \tag{25}$$

The function $\mathcal{P}_E(z,v)$ in its turn can be extracted from the correlation function

$$\mathcal{K}_{E,\Omega}(z_1,z_2) = \left\langle \rho_{E-\Omega/2}(z_1)\rho_{E+\Omega/2}(z_2)\right\rangle - \left\langle \rho_{E-\Omega/2}(z_1)\right\rangle\left\langle \rho_{E+\Omega/2}(z_2)\right\rangle \tag{26}$$

of the densities $\rho_E(z) = (1/M)\sum_{a=1}^{M}\delta(z - z_a(E))$ of eigenvalues $z_a(E)$ at a given energy E as:

$$\mathcal{P}_E(z,v) = M \lim_{\Omega\to 0} \Omega\,\mathcal{K}_{E,\Omega}(z_1 = z - v\Omega/2, z_2 = z + v\Omega/2) \tag{27}$$

Finally, to get access to the correlation function (26) we introduce the related object in terms of the traces of the resolvent:

$$f(z_1,z_2) = \left\langle \mathrm{Tr}\frac{1}{z_1 - i\epsilon - \hat{K}(E-\Omega/2)}\,\mathrm{Tr}\frac{1}{z_1 + i\epsilon - \hat{K}(E+\Omega/2)}\right\rangle, \tag{28}$$

from which the required correlation function can be extracted as

$$\mathcal{K}_{E,\Omega}(z_1, z_2) = \frac{1}{2\pi^2 M^2} \mathrm{Re}\, f_c(z_1, z_2), \quad f_c(z_1, z_2) = f(z_1, z_2) - f^-(z_1) f^+(z_2). \tag{29}$$

In order to calculate (28) we obviously need to specify the explicit form of the K-matrix. In the framework of the present approach $\hat{K}$ can be written in terms of the random Hamiltonian of a closed system represented by $L \times L$ Hermitian matrix $\hat{H}$, and some deterministic $L \times M$ matrix $\hat{W}$ describing the coupling of the system to the leads, see[5,49,50]:

$$\hat{K} = \hat{W}^\dagger \frac{1}{E - \hat{H}} \hat{W}. \tag{30}$$

An important combination of these two matrices is the so-called effective non-Hermitian Hamiltonian $\hat{H}_{eff} = \hat{H} - i\hat{\Gamma}$, where $\hat{\Gamma} = \pi \hat{W} \hat{W}^\dagger$. Actually, for the scattering problem $\hat{H}_{eff}$ in many respects replaces the Hamiltonian of the closed system $\hat{H}$.

There are several convenient microscopic models for the Hamiltonian $\hat{H}$ describing the quasi-one-dimensional disordered wire decoupled from leads. On may, for example, employ the one-dimensional variant[42] of the gauge-invariant N-orbital model introduced by Wegner[43]. Here we prefer to work within the ensemble of $L \times L$ Banded Random Matrices (BRM), see[7,44] for detail and discussions. In the limit of large number of orbitals $N \gg 1$ and of the large widths of the band $b \gg 1$ both microscopic models can be reduced to the same field-theoretical construction known as the nonlinear $\sigma-$model and are therefore equivalent. For the sake of simplicity in the main part of this section we consider explicitly the case of Hermitian matrices corresponding to systems with broken time reversal invariance due to a magnetic field. The calculation for the case of preserved time reversal invariance is similar in spirit, although more involved technically, and we quote the corresponding results in the very end.

The matrix elements of the Hamiltonian $H_{ij}, i \leq j$ are chosen to be independent random variables with the joint probability density

$$\mathcal{P}(H) = \mathcal{N} \prod_{i=1}^{L} \exp\left(-\frac{H_{ii}^2}{2V_{ii}}\right) \prod_{i<j} \exp\left(-\frac{|H_{ij}|^2}{2V_{ij}}\right), \tag{31}$$

where the variances V_{ij} decay rapidly outside a band of width $2b$ around the main diagonal:

$$V_{ij} = \frac{\lambda^2}{2b} \exp\left(-\frac{|i-j|}{b}\right) \tag{32}$$

This ensemble was studied intensively during last years and many analytical results are known in the limit $L \gg b \gg 1$ in the closed form[44]. As to the matrix $\hat{\Gamma} \equiv \pi \hat{W} \hat{W}^\dagger$ describing coupling to a single $M-$channel lead attached to the edge of the sample, it can be verified that in the limit $L \gg b \gg M$ that matrix can be chosen diagonal with only first M elements different from zero: $\hat{\Gamma} = \mathrm{diag}(\gamma_1, \ldots, \gamma_M, 0, \ldots, 0)$. The eigenvalues γ_a define the strength of coupling to a given channel in the lead, and will enter the final expression via the coupling coefficients $g_{\gamma_a} = 2/T_a - 1$, see Eq.(42) below. The perfect coupling corresponds as usual to $g_{\gamma_a} = T_a = 1$.

To find the correlation function (28) we express it via the generating function:

$$f(z_1, z_2) = \frac{\partial^2}{\partial J_1 \partial J_2} \left[\left(\frac{Z_J^{(1)} Z_J^{(2)}}{Z_{J=0}^{(1)} Z_{J=0}^{(2)}} \right)^M \mathcal{F}(J_1, J_2) \right] \Bigg|_{J_1 = J_2 = 0},$$

$$\mathcal{F}(J_1, J_2) = \left\langle \frac{\mathrm{Det}[E - \Omega/2 - H_{eff}(Z_J^{(1)})] \, \mathrm{Det}[E + \Omega/2 - H_{eff}(Z_J^{(2)})]}{\mathrm{Det}[E - \Omega/2 - H_{eff}(Z_{J=0}^{(1)})] \, \mathrm{Det}[E + \Omega/2 - H_{eff}(Z_{J=0}^{(2)})]} \right\rangle, \tag{33}$$

where we introduced the notations: $Z_J^{(p)} = z_p + (-1)^p \epsilon + J_p, \ p = 1, 2$ and defined the effective Hermitian matrices

$$H_{eff}(Z_J^{(p)}) = \hat{H} + \frac{1}{Z_J^{(p)}} \hat{\Gamma} \tag{34}$$

The ensemble averaging in Eq.(33) can be easily performed after one represent the determinants in terms of the Gaussian supersymmetric integrals[51]. We follow notations in[5], see also[7]:

$$\mathcal{F}(J_1, J_2) = (-1)^L \int [d\Psi] \exp\left\{ -iE\Psi^\dagger \hat{L}\Psi - i\frac{\Omega}{2}\Psi^\dagger \hat{L}\hat{\Lambda}\Psi + i\Psi^\dagger \hat{\Gamma} \otimes (\hat{L}\hat{U})\Psi \right\} \left\langle \exp\left\{ i\Psi^\dagger (\hat{H} \otimes \hat{L})\Psi \right\} \right\rangle, \tag{35}$$

where:

$$\Psi = \begin{pmatrix} \vec{\Psi}_1 \\ \vec{\Psi}_2 \end{pmatrix}, \quad \text{where} \quad \vec{\Psi} = \begin{pmatrix} \vec{S}_p \\ \vec{\chi}_p \end{pmatrix}, \quad p = 1, 2 \tag{36}$$

The elements of the L-dimensional vectors $\vec{S}_p$ and $\vec{\chi}_p$ are commuting and anticommuting variables respectively. The 4×4 matrices appearing in Eq.(35) are diagonal $\hat{L} = \text{diag}(1, 1, -1, 1)$, $\hat{\Lambda} = \text{diag}(1, 1, -1, -1)$, $\hat{U}^{-1} = \text{diag}(Z_{J=0}^{(1)}, Z_J^{(1)}, Z_{J=0}^{(2)}, Z_J^{(2)})$.

In order to calculate right-hand side of Eq.(35) we perform all the standard steps[7,44]: (i) average over the random matrices $\hat{H}$ according the distribution function (31); (ii) introduce auxiliary supermatrices $\hat{Q}_i$ that allow to decouple the obtained integral by the Hubbard-Stratonovich transformation; (iii) perform Gaussian integral over Ψ; (iv) employ the saddle-point approximation for the integrals over Q_i justified by large b and L. All these steps allow us to represent the main object of interest as:

$$f(z_1, z_2) = \lim_{J_{1,2} \to 0} \frac{\partial^2}{\partial J_1 \partial J_2} \int \prod_{i=1}^{L} d\mu(\hat{Q}_i) \, \exp\left[-\frac{\xi}{4} \sum_{i=1}^{L} \text{Str}\hat{Q}_i \hat{Q}_{i+1} - i\frac{\Omega}{2}\pi\nu \sum_{i=1}^{L} \text{Str}\hat{Q}_i \hat{\Lambda} \right] \prod_{a=1}^{M} f_a, \tag{37}$$

$$f_a = \text{Sdet}^{-1}\left[U^{-1} - \frac{\gamma_a}{V_0}\left(\frac{E}{2} - i\pi\nu\hat{Q}_a \right) \right], \quad V_0 = \sum_{i=1}^{L} V_{ij}. \tag{38}$$

Here the supermatrices $\hat{Q}_i$ satisfy the following constraints:

$$\hat{Q}_i^2 = \hat{I}, \quad \hat{Q}_i^\dagger = \hat{L}\hat{Q}_i\hat{L}, \quad \text{Str}\, \hat{Q}_i = 0. \tag{39}$$

The mean density of states ν around a point E in the spectrum is given in this limit by the semicircular law: $\nu = \frac{1}{2\pi V_0}\sqrt{4V_0 - E^2}$, whereas the parameter $\xi = ((\pi\nu V_0)^2 8b\lambda^{-2}e^{-1/b})/(1 - e^{-2/b}) \sim b^2$ is nothing else but the localization length of the wire detached from the leads[44]. The integral over supermatrices Q_i can be calculated using the transfer matrix technique[7,44]. The technical details of the calculation are rather similar to those performed in Refs[5,44], and we skip them here in favor of presenting the final result for the density of the (scaled) partial time delays $\tilde{\tau} = \tau/2\pi\nu$:

$$\mathcal{P}_{\tilde{\tau}}(\tilde{\tau}) = \frac{(-1)^{M+1}}{2\pi M!\, L^{M+1}\, \tilde{\tau}^{M+2}} \int_0^{2\pi} d\theta \, \left(g_\gamma - \sqrt{g_\gamma^2 - 1}\cos\theta \right)^M w_L^{(M+1)}\left(\frac{g_\gamma - \sqrt{g_\gamma^2 - 1}\cos\theta}{\tilde{\tau}L} \right), \tag{40}$$

Here the notation $w_L^{(M+1)}(z) \equiv \frac{d^{M+1}w_L(z)}{dz^{M+1}}$ stands for $(M+1)$-th derivative of the function $w_L(z)$ with respect to its argument. The function $w_L(z)$ is expressed in terms of the solution of the following differential equation:

$$\frac{\partial W}{\partial t}(y, t) = \left(y^2 \frac{\partial^2}{\partial y^2} - y \right) W(y, t), \quad W(y, 0) = 1 \tag{41}$$

as $w_L(z) = W(\xi z, L/\xi)$,. In deriving the expression Eq.(40) we assumed for simplicity that all the scattering channels have equal coupling strength $\gamma_a = \gamma$ related to the parameter g_γ as:

$$g_\gamma = \frac{1}{2\pi\nu\gamma}\left(\frac{\gamma^2}{V_0} + 1 \right). \tag{42}$$

For the important case of perfect coupling to leads $g_\gamma = 1$, see the discussion around Eq.(57), and the expression (40) can be considerably simplified:

$$\mathcal{P}_{\tilde{\tau}}(\tilde{\tau}) = \frac{(-1)^{M+1}}{M!\, L^{M+1}\, \tilde{\tau}^{M+2}} w_L^{(M+1)}\left(\frac{1}{\tilde{\tau}L} \right). \tag{43}$$

According to the results of Ref.[52] the distribution of the eigenfunction intensities $y = V|\psi_n(\mathbf{r})|^2$ at the edge of the sample can be expressed in terms of the same function $w_L(z)$ as:

$$\mathcal{P}_y(y) = \frac{1}{L^2} w_L^{(2)}\left(\frac{y}{L} \right) \tag{44}$$

Comparing these two expressions we conclude that

$$\mathcal{P}_{\tilde{\tau}}(\tilde{\tau}) = \frac{(-1)^{M+1}}{M!\,\tilde{\tau}^{M+2}}\,\mathcal{P}_y^{(M-1)}\left(\frac{1}{\tilde{\tau}}\right) \tag{45}$$

In particular for $M = 1$ we rediscover Eq.(11) verifying this relation for a quasi-one-dimensional geometry in an independent way. The exact solution of Eq.(41) can be written in terms of the modified Bessel functions[44] and for a single-channel setup it reproduces the result obtained by one of the authors in a different way[36].

Let us briefly discuss some asymptotic behaviour of the found solution in a few important limiting cases. For the case of a short wire, i. e. $L/\xi \ll 1$ the solution of Eq.(41) can be approximated by $W(y,t) \approx e^{-yt}$ so that $w_L(z) = e^{-Lz}$. This approximation yields the probability density for delay times

$$\mathcal{P}_{\tilde{\tau}}(\tilde{\tau}) = \frac{1}{M!\,\tilde{\tau}^{M+2}}\,e^{-1/\tilde{\tau}}, \tag{46}$$

which coincides with the distribution for "zero-dimensional" case derived in the framework of the standard RMT[5,11,12]. Expanding the exact solution in powers of the small parameter $1/g = L/\xi$ proportional to the inverse dimensionless conductance for quasi one-dimensional sample. We find:

$$\mathcal{P}_{\tilde{\tau}}(\tilde{\tau}) = \frac{1}{M!\,\tilde{\tau}^{M+2}}\,e^{-1/\tilde{\tau}}\left(1 + \frac{1}{3g}\left[M(M+1) - 2(M+1)\frac{1}{\tilde{\tau}} + \frac{1}{\tilde{\tau}^2}\right] + \ldots\right). \tag{47}$$

For $M = 1$ this formula gives back the expression Eq.(17), after we use that the value of the diffusion propagator κ is exactly equal to $2/3g$ at the edge of the quasi-one-dimensional sample. The relation holds for not too small values of delay times $\tilde{\tau} > L/\xi$. In the opposite limit the distribution is dominated by anomalously localized states[30] and has the following form:

$$\mathcal{P}_{\tilde{\tau}}(\tilde{\tau}) \sim \frac{1}{\tilde{\tau}^{(M+3)/2}}\exp\left(-4\sqrt{\frac{\xi}{L\tilde{\tau}}}\right) \tag{48}$$

In the case of a long enough wire $(L \gg \xi)$ the probability density function is determined by the stationary (t-independent) solution of Eq.(41) and is given by

$$\mathcal{P}_{\tilde{\tau}}(\tilde{\tau}) \approx \frac{(-1)^{M+1}}{M!\,\tilde{\tau}^{M+2}}\frac{8\xi^2}{L^2}\frac{\partial^{(M-1)}}{\partial\left[2\sqrt{\xi/(L\tilde{\tau})}\right]^{(M-1)}}\left[K_1^2\left(2\sqrt{\xi/(L\tilde{\tau})}\right) + K_0^2\left(2\sqrt{\xi/(L\tilde{\tau})}\right)\right]. \tag{49}$$

It has the same asymptotic behavior as Eq.(48).

Finally let us briefly discuss the case of the Hamiltonian matrix $\hat{H}$ real and symmetric, which is pertinent for the systems respecting the time-reversal invariance. It turns out that the above derivation can be straightforwardly generalized to this case. We omit all the cumbersome details of intermediate calculation and proceed directly to the final result for the density of partial time delays for M equivalent channels and zero energy $E = 0$:

$$\mathcal{P}_{\tilde{\tau}}(\tilde{\tau}) = \frac{C_M}{\tilde{\tau}^2}\int_{-\infty}^{\infty}dz\,\frac{1}{1+z^2}\int_{-\infty-i0}^{\infty-i0}dx\,\frac{w_L^{(2)}(ix)}{\left[1 - \frac{2i\gamma L(z^2+1)\tilde{\tau}x}{z^2+\gamma^2}\right]^{M/2}}, \tag{50}$$

Here we use the same notation as in Eq. (40) and C_M is a normalization constant, with the infinitesimal shift $-i0$ ensuring the integral to be well-defined. The expression can be again simplified for the perfect coupling ($\gamma = 1$):

$$\mathcal{P}_{\tilde{\tau}}(\tilde{\tau}) = \frac{\pi C_M}{\tilde{\tau}^2}\int_{-\infty-i0}^{\infty-i0}dx\,\frac{w_L^{(2)}(ix)}{[1 - 2iL\tilde{\tau}x]^{M/2}}, \tag{51}$$

We shall consider separately the case of even and odd channel number M. For $M = 2p$, with p being an integer we rewrite the above expression as:

$$\mathcal{P}_{\tilde{\tau}}(\tilde{\tau}) = \frac{\pi C_p}{\tilde{\tau}^{2+p}}\frac{(-1)^{p-1}}{(p-1)!(2L)^p}\frac{d^p}{dz^p}F(z)|_{z=\frac{1}{2L\tilde{\tau}}}, \quad F(z) = \int_{-\infty-i0}^{\infty-i0}dx\,\frac{w_L^{(2)}(ix)}{z - ix} \tag{52}$$

One can show that the function $w_L^{(2)}(ix)$ is analytic in the lower half plane of the complex variable ix, and therefore the integral in (52) is given by the residue at the pole $x = -\frac{i}{2L\tilde{\tau}}$. Using the properties $w_L(0) = 1$ and $w_L(\infty) = 0$ to fix the normalization constant, we finally arrive at the following simple expression:

$$\mathcal{P}_{\tilde{\tau}}(\tilde{\tau}) = \frac{1}{\tilde{\tau}^{2+p}} \frac{(-1)^{p-1}}{p!(2L)^{p+1}} \frac{d^{p+1}}{dz^{p+1}} w_L(z)|_{z=\frac{1}{2L\tilde{\tau}}} \tag{53}$$

generalizing Eq. (43) to the case of preserved time-reversal invariance, and even number of channels.

For the odd number of channels $M = 2p + 1$ we use a very similar manipulation and rewrite (51) as:

$$\mathcal{P}_{\tilde{\tau}}(\tilde{\tau}) = \frac{\pi C_p}{(2L)^{\frac{M}{2}} \tilde{\tau}^{2+\frac{M}{2}}} \frac{(-2)^p}{(2p-1)!!} \frac{d^p}{dz^p} F(z)|_{z=\frac{1}{2L\tilde{\tau}}}, \quad F(z) = \int_{-\infty-i0}^{\infty-i0} dx \frac{w_L^{(2)}(ix)}{\sqrt{z-ix}} \tag{54}$$

The integral featuring in this expression can be reduced to that along the branch cut parameterized as $x = -iq - \frac{i}{2L\tilde{\tau}}$, where $0 \leq q < \infty$. After restoring the normalization constant we find:

$$\mathcal{P}_{\tilde{\tau}}(\tilde{\tau}) = \frac{(-1)^p}{2^{p+3/2}\sqrt{\pi}\Gamma(p+3/2)} \frac{1}{L^{p-\frac{1}{2}}\tilde{\tau}^{p+\frac{5}{2}}} \frac{d^p}{dz^p} F(z)|_{z=\frac{1}{2L\tilde{\tau}}}, \quad F(z) = \int_0^\infty \frac{dq}{\sqrt{q}} w_L^{(2)}(q+z) \tag{55}$$

Since the one-channel case corresponds to $p = 0$ we should expect that the corresponding distribution of time delays is related by Eq.(11) to known distribution of eigenfunction intensities found in[52], see also Eq.(3.14) of[30]. It is easy to see that this is indeed the case. For arbitrary $p > 0$ the time delay density can be also related to eigenfunction statistics by differentiation, similarly to Eq.(45). Finally, for the case of a short wire we can use again the approximate solution of the differential equation (41) $W(y,t) \approx e^{-yt}$, so that $w_L(z) = e^{-Lz}$ and the integral in Eq.(51) can be calculated straightforwardly for all M, both odd and even. This yields the density of partial time delays well-known from the standard RMT[13]:

$$\mathcal{P}_{\tilde{\tau}}(\tilde{\tau}) = \frac{1}{2^{(M/2+1)}\Gamma(M/2+1)\tau^{M/2+2}} \exp\left[-\frac{1}{2\tilde{\tau}}\right] \tag{56}$$

IV. DISCUSSION AND CONCLUSIONS.

The main message of this paper is that measuring scattering characteristics such as time delays one can get a direct access to properties of eigenfunctions of the closed counterpart of the scattering system. For the simplest case of the single-channel reflection from a disordered/chaotic sample the relation is most explicit and is summarized in Eq.(11), see also Eq.(57) below. It is very general (see discussion below), and is valid for all pure symmetry classes (and even in the crossover regimes, see Eqs.(14)) and in the whole parameter region ranging from fully localized to delocalized eigenstates via the critical region of the Anderson transition.

In fact, one could suspect the existence of some relation of this sort already from the original derivation[12] of time delays distribution for the simplest case of zero spatial dimension (RMT regime), see Eq.(13). Indeed, the derivation was based on relating the time delay statistics to that of residues of the K-matrix by employing the so-called Wigner conjecture. According to RMT those residues are chi-squared distributed as a consequence of eigenfunction fluctuations Eq.(12). However, the authors[12] failed to trace the generality, broad validity and physical consequences of the ensuing relation beyond that simplest case.

The problem of relating scattering characteristics to the properties of closed systems also enjoyed some discussion in the framework of a discrete-time evolution of the so-called network models for disordered electron transport, see e.g.[54] for discussion and further references. Those models do not use a Hamiltonian as the starting point, but rather operate with networks of unidirectional links serving originally to mimic electron propagation in strong magnetic field of Quantum Hall phenomenon. After the present paper had already been submitted for publication, we were informed by J.T.Chalker that a relation analogous to our Eq.(11) between the current density in a link and the energy derivative of the total phaseshift emerged in a one-dimensional version of the network model considered in[55].

Let us briefly emphasize a few points related to the validity of the central formula Eq.(11) which may need clarification. First of all, statistics of the eigenfunction intensities entering that relation refers to the position **r** of the lead/antenna port, so that both quantities should be considered locally. This may be important if the antenna attached not to the bulk, but rather to the boundary of the sample, where eigenfunction statistics may be modified due to boundary effects, see e.g.[30]. Second, the derivation given in the sec. II of the paper implies two very general assumptions only: i) The energy E of the spectrum is not a singular point of the density of states, so that the latter can be approximated by the constant value in the energy window of the order of the mean level spacing. This

condition is violated e.g close to the spectral edges, or close to the band centre for systems with special (e.g. "chiral", see[18] and references therein) symmetries; and ii) The phase of the scattering matrix is statistically independent from the modulus of the scattering matrix and is uniformly distributed over the unit circle[46]. The latter property is a very general feature of the *perfect coupling* regime defined as one for which the energy/ensemble average $\langle S \rangle$ of the S-matrix vanishes. General case of non-perfect coupling is characterized by the so-called transmission coefficient $T = 1 - |<S>|^2 < 1.$[49,50], and it is well-known that statistics of various quantities at the perfect coupling $T = 1$ allows one to find the properties for a general case $T \leq 1$ after simple manipulations. In particular, such a relation for the partial delay times at arbitrary coupling T can be found in[16], see also[15]. It allows one to write down the expression for the delay time distribution for any coupling in terms of the statistics of the eigenfunction intensities as (cf. Eq.(40)):

$$\mathcal{P}_w(\tilde{\tau}_w) = \frac{1}{2\pi} \int_0^{2\pi} d\theta \, \frac{1}{2\pi\rho(\theta)\tilde{\tau}_w^3} \mathcal{P}_y \left(\frac{1}{2\pi\rho(\theta)\tilde{\tau}_w} \right), \tag{57}$$

where the density of scattering phases $\rho(\theta) = [2\pi(g_\gamma - \sqrt{g_\gamma^2 - 1}\cos(\theta))]^{-1}$ is characterized by the parameter g_γ[53] related to the coupling strength $g_\gamma \equiv 2/T - 1$. For the relation of the parameter g_γ to "microscopic" characteristics of the banded random matrix model see Eq.(42) of the present paper. In particular, the relation (57) implies that moments of the inverse time delay are always proportional to the moments of the eigenfunction intensity $y = V|\psi_n(\mathbf{r})|^2$ as:

$$\langle \tilde{\tau}_w^{-k} \rangle = \langle y^{k+1} \rangle \frac{1}{2\pi} \int_0^{2\pi} d\theta \, [2\pi\rho(\theta)]^{k+1} . \tag{58}$$

The range of validity of the relations Eqs.(57, 58) for a given microscopic model of a disordered system is the same as the range of validity of the nonlinear $\sigma-$model description of the latter, provided the singularities in spectrum are avoided.

For the case of more than one channel statistics of *partial* time delays can be also related to the statistics of eigenfunctions as is clear e.g. from Eq.(45). Here, however we so far were unable to provide the generality and clarity achieved for the single-channel derivation. We also leave as an interesting open question in which way the *proper* time delays whose statistics was in much detail investigated in[14] could be related to statistics of eigenfunctions. The same question remains for the Wigner time delay for more than one open channel.

Referring to critical regime and corresponding multifractality as reflected in the statistics of time delays, let us mention that recently the scaling of the second negative moment of the Wigner delay time was studied numerically at the metal-insulator transition[56]. It was indeed found that the scaling is anomalous, but in contrast to the behaviour $\langle \tau_w^{-2} \rangle \sim L^{-2D_3}$ expected from (20) the authors reported $\langle \tau_w^{-2} \rangle \sim L^{-D_2}$. Precise reasons for such discrepancy are not clear for us at the moment and deserve a separate investigation. We would like only to mention that (i) for the case of $3D$ Anderson model the Wigner time delay was numerically calculated for a disordered sample coupled to a very large number of channels $M \gg 1$. This limiting case is not covered by the present work, and requires separate consideration; (ii) it is an open question to which extent the numerical value $D_2 = 1.7$ reported in[56] can be used for reliable comparison with eigenfunction properties. Indeed, it differs considerably from the value $D_2 = 1.3 \pm 0.05$ found in[28] after a careful data analysis on eigenfunction statistics. Another point that deserves mentioning is that analytical results for the anomalous scaling of the eigenfunctions were obtained in earlier publications after performing the spatial averaging over the whole sample. Thus one expects validity of Eqs. (21), (22) after averaging the left-hand side over all possible positions of the lead/antenna. Alternatively, one must be sure that the point where the lead is attached is a "representative" one, meaning that the local statistics at this point is the same as the global one. The last condition is usually satisfied due to eigenfunction ergodicity, but some special situations like a lead attached to the boundary of the sample may require special care.

---

* Electronic address: aossipov@ictp.trieste.it

† Electronic address: yan.fyodorov@brunel.ac.uk

[1] E. P. Wigner, Phys. Rev. **98**, 145 (1955)

[2] C. A. A. de Carvalho and H. M. Nussenzveig, Phys. Rep. **364**, 83 (2002).

[3] G. Iannaccone Phys. Rev. B **51**, 4727 (1995)

[4] F. T. Smith, Phys. Rev. **118**, 349 (1960)

[5] Y. V. Fyodorov and H.-J. Sommers, J. Math. Phys. **38**, 1918 (1997).

[6] M.Bauer, P.A.Mello and K.M.Voy, Z. Physik A **283**, 151 (1979); V.L.Lyuboshitz, JETP Lett. **28**, 30 (1978); Sov.J. Nucl.Phys. **27**, 502 (1978) and Phys.Lett. B **72**, 41 (1988); J. G. Muga and D. M. Wardlaw, Phys. Rev. E **51**, 5377 (1995)

[7] F. Haake, *Quantum signatures of chaos*, Springer-Verlag, Berlin, 2001.

[8] H.-J. Stökmann, *Quantum Chaos: An Introduction*, Cambridge University Press, Cambridge, 1999.

[9] T. Guhr, Müller-Groeling, and H. A. Weidenmüller Phys. Rep. **299**, 189 (1998).

[10] N. Lehman, D. V. Savin, V. V. Sokolov, and H.-J. Sommers, Physica D **86**, 572 (1995)

[11] Y. V. Fyodorov and H.-J. Sommers, Phys. Rev. Lett. **76**, 4709 (1996)

[12] V. A. Gopar, P. A. Mello, and M. Büttiker, Phys. Rev. Lett. **77**, 3005 (1996).

[13] Y. V. Fyodorov, D. V. Savin, and H.-J. Sommers, Phys. Rev. E **55**, R4857 (1997).

[14] P. W. Brouwer, K. M. Frahm, and C. W. J. Beenakker, Phys. Rev. Lett. **78**, 4737 (1997); Waves Random Media **9**, 91 (1999); H. Schomerus, K. J. H. Van Bemmel, and C. W. J. Beenakker, Europhys. Lett. **52**, 518 (2000).

[15] V.A.Gopar and P.A.Mello, Europh.Lett.**42**, 131 (1998)

[16] D. V. Savin, Y. V. Fyodorov, and H.-J. Sommers, Phys. Rev. E **63**, 035202(R) (2001); H.-J. Sommers, D. V. Savin, and V. V. Sokolov, Phys. Rev. Lett. **87**, 094101 (2001).

[17] D. V. Savin and H.-J. Sommers, Phys. Rev. E **68**, 036211 (2003).

[18] Y. V. Fyodorov and A. Ossipov, Phys. Rev. Lett. **92**, 084103 (2004).

[19] M. Glück, A. R. Kolovsky, and H. J. Korsch, Phys. Rev. Lett. **82**, 1534 (1999); T. Kottos and U. Smilansky, J. Phys. A: Math. Gen. **36**, 3501 (2003); G. B. Akguc and L. E. Reichl, Phys. Rev. E **67**, 046202 (2003) and *ibid.* **64**, 056221 (2001).

[20] B. Eckhardt, Chaos **3**, 613 (1993); R.O. Vallejos, A. M. O. de Almeida, and C.H. Lewenkopf, J. Phys. A: Math. Gen. **31**, 4885 (1998); C. H. Lewenkopf and R. O. Vallejos, Phys. Rev. E **70**, 036214 (2004) and J. Phys. A: Math. Gen. **37**, 131 (2004).

[21] H.-J. Stöckmann and J. Stein, Phys. Rev. Lett. **64**, 2215 (1990); H.-D. Graf, H. Harney, H. L. Lengeller et al., Phys. Rev. Lett. **69**, 1296 (1992); J. Stein, H.-J. Stöckmann, and U. Stoffregen, Phys. Rev. Lett. **75** 53 (1995); C. Ellegaard, T.Guhr, K. Lindemann et al., Phys. Rev. Lett. **75**, 1546 (1995); D. H. Wu, J. S. A. Bridgewater, A. Gokirmak, and S. M. Anlage, Phys. Rev. Lett. **81**, 2890 (1998); O. I. Lobkis, I. S. Rozhkov, and R. L. Weaver, Phys. Rev. Lett. **91**, 194101 (2003).

[22] B. L. Altshuler and B. I. Shklovski, Sov. Phys. JETP **64**, 127 (1986); V. E. Kravtsov and A. D. Mirlin, JETP Lett. **60**, 656 (1994); A. G. Aronov, V. E. Kravtsov, and I. V. Lerner, Phys. Rev. Lett. **74**, 1174 (1995); J. T. Chalker, V. E. Kravtsov, and I. V. Lerner JETP Letters **64**, 386 (1996).

[23] V. I. Falko and K. B. Efetov, Europhys. Lett. **32**, 627 (1995); V. I. Falko and K. B. Efetov, Phys. Rev. B **52**, 17413 (1994).

[24] Y. V. Fyodorov and A. D. Mirlin, JETP Lett. **60**, 790 (1994); Y. V. Fyodorov and A. D. Mirlin, Phys. Rev. B **51**, 13403 (1995).

[25] Y. M. Blanter and A. D. Mirlin, Phys. Rev. E **55**, 6514 (1997).

[26] V. N. Prigodin, Phys. Rev. Lett. **74**, 1566 (1996); V. N. Prigodin and B. L. Altshuler, Phys. Rev. Lett. **80**, 1944 (1998); I. E. Smolyarenko and B. L. Altshuler, Phys. Rev. B **55**, 10451 (1997).

[27] K. Muller, B. Mehlig, F. Milde et al., Phys. Rev. Lett. **78**, 215 (1997); V. Uski, B. Mehlig and M. Schreiber, Phys. Rev. B, **66**, 233104 (2002); B. K. Nicolic and V. Z. Cerovsky , Eur. Phys. J. B **30**, 227 (2002); A. Ossipov, T. Kottos, and T. Geisel, Phys. Rev. E, **65**, 055209(R) (2002).

[28] A. Mildenberger, F. Evers, and A. D. Mirlin, Phys. Rev. B **66**, 033109 (2002).

[29] A. Kudrolli, V. Kidambi, and S. Sridhar, Phys. Rev. Lett. **75**, 822 (1995); C. Dembowski, H. D. Graf, R. Hofferbert et al., Phys. Rev. E, **60**, 3942 (1999); H.-J. Stöckmann, M. Barth, U. Dorr et al., Physica E. **9**, 571 (2001); P. Pradhan and S. Sridhar, Pramana-J. Phys. **58**, 333 (2002).

[30] A. D. Mirlin, Phys. Rep. **326**, 259 (2000).

[31] A. M. Jayannavar, G. V. Vijayagovindan, and N. Kumar, Z. Phys. B **75**, 77 (1989); C. Texier and A. Comtet, Phys. Rev. Lett. **82**, 4220 (1999); M.Titov and Y.V.Fyodorov, Phys. Rev. B **61**, R2444(2000); S.A. Ramakrishna and N. Kumar, Phys. Rev. B **61**, 3163(2000) and Eur. Phys. J. B **23**, 515 (2001); A. Ossipov, T. Kottos, and T. Geisel, Phys. Rev. B **61**, 11411 (2000).

[32] H. Schomerus, Phys. Rev. E **64**, 026606 (2001).

[33] F. Steinbach, A. Ossipov, T. Kottos, and T. Geisel, Phys. Rev. Lett. **85**, 4426 (2000).

[34] T. Kottos and M. Weiss, Phys. Rev. Lett. **89**, 056401 (2002).

[35] A. Ossipov, T. Kottos, and T. Geisel, Europhys. Lett. **62**, 719 (2003); T. Kottos, A. Ossipov and T. Geisel, Phys. Rev. E, **68**, 066215 (2003).

[36] Y. V. Fyodorov, JETP Lett. **78**, 250 (2003)

[37] A. Z. Genack, P. Sebbah, M. Stoytchev, and B. A. van Tiggelen, Phys. Rev. Lett. **82**, 715 (1999); A. A. Chabanov and A. Z. Genack, Phys. Rev. Lett. **87**, 233903 (2001).

[38] Z. Jian, J. Pearce, and D. M. Mittleman, Phys. Rev. Lett. **91**, 033903 (2003); J. Pearce, Z. Jian, and D. M. Mittleman, Phys. Rev. Lett. **91**, 043903 (2003).

[39] R. A. Méndez-Sánchez *et al.*, Phys. Rev. Lett. **91**, 174102 (2003); U. Kuhl , M. Martínez-Mares, R. A. Méndez-Sánchez,

and H.-J. Stöckmann, *e-print* cond-mat/0407197.

[40] E. Doron, U.Smilansky, and A. Frenkel **65**, 3072 (1990); C. W. J. Beenakker and P.W. Brouwer, Physica E (Amsterdam) **9**, 463 (2001).

[41] There is a misprint in the formula Eq.(5) of[13], the correct factor in that formula should be $\lambda_2^{(M+3)/2}$ rather than $\lambda_2^{(M+3/2)}$.

[42] S.Iida, H. A. Weidenmüller and J. A. Zuk, Ann. Phys. (NY) **200**, 219 (1990).

[43] L. Schäfer and F. Wegner, Z. Phys. B **38**,113 (1980).

[44] Y. V. Fyodorov and A. D. Mirlin, Int. J. Mod. Phys. B **8** 3795 (1994); Y. V. Fyodorov and A. D. Mirlin, Phys. Rev. Lett. **67**, 2405 (1991).

[45] K. B. Efetov and A. I. Larkin, Zh. Eksp. Teor. Fiz. **85**, 764 (1983) [Sov. Phys. JETP **58**, 444 (1983)].

[46] Y. V. Fyodorov and D.V. Savin, Pis'ma v Zh. Eksp. Teor. Fiz. **80** , 855 (2004) [JETP Letters **80**, 725 (2004)].

[47] V. I. Falko and K. B. Efetov, Phys. Rev. B **50**, 11267 (1994); see also H.-J.Sommers and S. Iida , Physical Review E **49**, R2513 (1994).

[48] A. D. Mirlin, Y. V. Fyodorov, F.-M. Dittes, J. Quezada, and T. H. Seligman, Phys. Rev. E **54**, 3221 (1996); A. D. Mirlin and F. Evers, Phys. Rev. B **62**, 7920 (2000).

[49] J. J. M. Verbaarschot, H. A. Weidenmüller, and M. R. Zirnbauer, Phys. Rep. **129**, 367 (1985).

[50] V. V. Sokolov and V. G. Zelevinsky, Nucl. Phys. A **504**, 562 (1989).

[51] K. B. Efetov , Supersymmetry in Chaos and Disorder, (Cambridge University Press, 1997)

[52] A. D. Mirlin and Y. V. Fyodorov, J. Phys. A: Math. Gen. **26**, L551 (1993).

[53] The parameter g_γ used in this paper was denoted as g in[5] and many publications on chaotic quantum scattering. Here we have to change the notation to avoid confusion with the dimensionless conductance.

[54] R.Klesse and M.R.Zirnbauer, Phys.Rev.Lett. **86**, 2094 (2001); M.Janssen, M. Metzler and M.R. Zirnbauer Phys.Rev.Lett. **59**, 15836 (1999)

[55] J. Chalker, S. Siak , J.Phys.Cond.Matt. **2**, 2671 (1990)

[56] J. A. Méndez-Bermédez and T. Kottos, *e-print* cond-mat/0401098 and private communication.